Name_________________________

CS 383
Exam 1 Solutions

There are 6 numbered questions. The 6 parts of Question 1 are worth 4 points each. Questions 2 through 6 are worth 15 points each. You get one point for free.

1. Which languages are regular? You don't need to prove your answers. Write an "R" in the blank next to the description of each language you think is regular. Write "N" for any language you think is not regular. In each case the alphabet is $\Sigma=\{0,1\}$

   a. __**R**__Strings that end in exactly five 1s. So 01011111 is in this language but 010111111 is not.

   b. __**R**__Strings with any number of 0s followed by an even number of 1s.

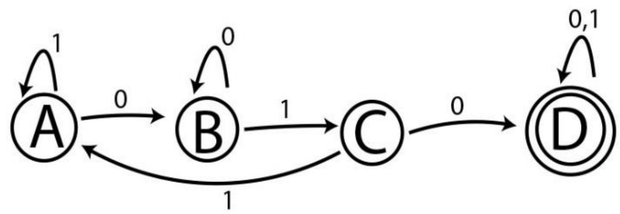
   c. __**R**__ $\{0^m1^n$ | if m is even then n is also even; if m is odd then n is also odd$\}$

   d. __**R**__Strings where the digits sum to a number divisible by 5 (i.e., the digits sum to 0, 5, 10, 15, etc.)

   e. __**N**__Strings where there are at least as many 0s as 1s.

   f. __**R**__$0^*\mathcal{L}$ where $\mathcal{L} = \{0^n|$ n is prime $\}$. Note that strings in this language have any number of 0s followed by a prime number of 0s.

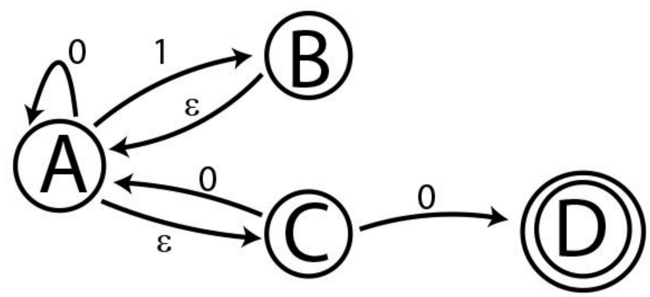
2. Give a DFA for the strings of 0s and 1s that contain the substring 010. For example, 110101 should be accepted by this DFA but 1001100 should not be accepted.



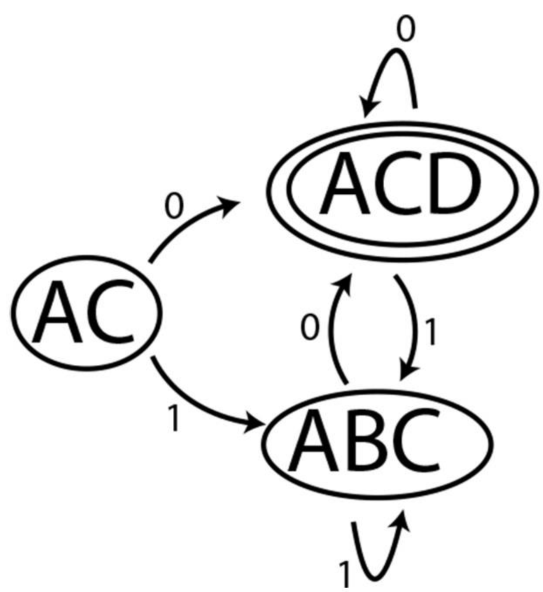
3. Here is an ε-NFA, with start state A.

a) Convert this NFA to a DFA

b) Describe in English the strings it accepts.



Solution:

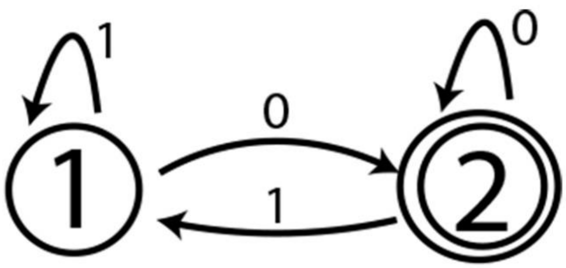


This accepts all strings ending in 0.

4. Suppose we know that for some language $\mathcal{L}$ the language $00\mathcal{L} = \{00\alpha \mid \alpha \epsilon\ \mathcal{L}\}$ is regular. Must $\mathcal{L}$ be regular? Either give an example where $\mathcal{L}$ is not regular and $00\mathcal{L}$ is regular, or else show that $\mathcal{L}$ must be regular if $00\mathcal{L}$ is.

The language $\mathcal{L}$ must be regular. Suppose $P = (\Sigma,Q,\delta,s,F)$ is a DFA accepting $00\mathcal{L}$. Let $q = \delta(s, 0)$ and let $q1=\delta(q,0)$. State q1 is where you get to in P on input 00. Let $P'= (\Sigma,Q,\delta,q1,F)$. P' is the same as P only with start state q1. Now suppose string α is in $\mathcal{L}$. Then 00α is in 00L and takes P from state s to q to q1 and then eventually to a final state. So α takes P' from q1 to a final state, and P' accepts α. Similarly, if α takes P' from q1 to a final state then 00α takes P from s to a final state, so 00α is in $00\mathcal{L}$ and α must be in $\mathcal{L}$. Altogether, the DFA P' accepts α if and only if α is in $\mathcal{L}$, so $\mathcal{L}$ is regular.

5. Consider the following DFA. We had an algorithm for converting a DFA to a regular expression. This involved making a table of regular expressions r_{ij}^{k} .



Here is the first column of a table of the r_{ij}^{k} expressions; find the 4 entries of the second column.

	k=0	k=1
r_{11}^{k}	ε+1	**1***
r_{12}^{k}	0	**1*0**
r_{21}^{k}	1	**11***
r_{22}^{k}	ε+0	**ε+1*0**

$$\boldsymbol{r_{ij}^{1} = r_{ij}^{0}+r_{i1}^{0}(r_{11}^{0})^{*}r_{1j}^{0}}$$

6. Use the pumping lemma to show carefully that the language $\{0^m1^n0^n \mid m>=2, n >=0\}$ is not regular.

   Suppose this language is regular; let p be its pumping constant. Let w = $0^21^p0^p$. This is longer than p, so let w=xyz be any decomposition of w where y is not empty and $|xy|<p$. All of y must come from the initial 0^21^p elements of w. If y contains any initial 0s then xy^0z has fewer than 2 initial 0s. If y contains any 1s then xy^0z has fewer 1s than trailing 0s. Either way, xy^0z is not an element of our language so our string w is not pumpable. This contradicts the Pumping Lemma, so our language can't be regular.